# تقنية قوية لوضع العلامات المائية على الصور ذات التدرج الرمادي العمياء استناداً إلى تحلل شور وتحليل الإنتروبيا

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مستخلص:

تم تقديم طريقة جديدة قوية للعلامة المائية للصور العمياء تعتمد على تحويل المويجات الرفيعة، وتحلل شور بالاشتراك مع تحويل جيب التمام المنفصل، وتحليل الإنتروبيا لحماية حقوق الطبع والنشر لمعلومات الوسائط المتعددة، في هذه الورقة. في البداية، يتم تطبيق تحويل المويجات الرفعية ثنائي المستوى على الصورة الأصلية لتحسين عدم رؤية طريقة العلامة المائية ومن ثم يتم تحليل النطاق الفرعي عالي التردد لتحويل المويجات الرفعية ثنائي المستوى بواسطة تحويل جيب التمام المنفصل .بعد ذلك، يتم تقسيم معاملات تحويل جيب التمام المنفصل إلى كتل غير متداخلة 4×4. بعد ذلك، يتم تطبيق تحلل شور على كل كتلة محددة، بينما يتم استخدام الصف الأول، عنصر العمود الأول من المصفوفة المثلثية العليا لتضمين العلامة المائية. للتقييم، يتم استخدام اختفاء ومتانة طريقة العلامة المائية المقترحة، ونسبة الذروة للإشارة إلى الضوضاء، والارتباط المتبادل الطبيعي لقياس جودة وقدرة طريقة العلامة المائية المقترحة على أن تكون قوية ضد الإشارة عمليات المعالجة والهجمات الهندسية. وقد أظهرت النتائج التجريبية أن الطريقة المقترحة قد حققت مقايضة جيدة للغاية بين الخفاء والمتانة. أظهرت المقارنات مع المخططات الأخرى أن نظام العلامات المائية الرقمية المقترح يتمتع بأداء متفوق من حيث الاختفاء عن غيره.

# A Robust Blind Grayscale Image Watermarking Technique Based on Schur Decomposition and Entropy Analysis

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## Abstract

A new robust blind image watermarking method based on lifting wavelet transform (LWT), and Schur decomposition in combination with discrete cosine transform (DCT), and entropy analysis for copyright protection of multimedia information is introduced in this paper. At first, the 2-level LWT is applied to the original image to improve the invisibility of the watermarking method and then the high-frequency sub-band of 2-level LWT is decomposed by DCT. Then, the DCT coefficients are divided into 4×4 non-overlapping blocks. After that, Schur decomposition applies to each selected block, while the first row, the first column element of the upper triangular matrix is used to embed the watermark. For evaluation, the invisibility and robustness of the proposed watermarking method, the peak signal to noise ratio (PSNR), and normalized cross-correlation (NC) are used to measure the quality and the ability of the proposed watermarking method to be robust against signal processing operations and geometric attacks. Experimental results have demonstrated that the proposed method is achieved a very good tradeoff between invisibility and robustness. The comparisons with other scheme have shown that the proposed digital watermarking scheme has a superior performance in terms of invisibility than other.

**Keywords** discrete cosine transform, watermarking, entropy analysis, Schur decomposition, lifting wavelet transform.

## Introduction

Recently, multimedia information distribution (video, image, text, audio) on the internet can be easily redistributed, altered, and copied. Nowadays, the copyright protection of this multimedia information has become the most pressing problem [1]. Therefore, several efficient copyright protection techniques have been proposed to protect multimedia information such as steganography, encryption, and watermarking [2]. In particular, digital watermarking is a useful technology for hiding information (watermark) into other information [3]. In addition, digital watermarking techniques are utilized in various applications such as content identification, broadcast monitoring, fingerprinting, tamper detection, and content authentication [4]. The hiding and extraction of multimedia information can be done in transform and spatial domains. The hiding information in the spatial domain has been commonly utilized for authentication and tamper detection. This technique can provide a high level of invisibility, whereas the hidden information can be destroyed under several image processing and geometrical attacks. Hiding multimedia information techniques in the transform domain can achieve high resistance to several image processing and geometrical attacks. [5]. discrete wavelet transform (DWT) [6, 7], Fourier transform [8, 9], discrete cosine transform (DCT) [10, 11], Hadamard transform [12, 13], and matrix decompositions such as QR decomposition [14, 15], Hessenberg decomposition [16, 17], and singular value decomposition (SVD) [18, 19], etc.

A novel robust blind digital grayscale image watermarking method for copyright protection based on discrete cosine transform, Schur decomposition, lifting wavelet transform (LWT), and entropy analysis is proposed in this paper. Here, a 2-level lifting wavelet transform is applied to the original image to obtain the coefficients of a high-frequency subband which is then transformed by discrete cosine transform. Afterward, dividing the coefficients of discrete cosine transform into non-overlapping blocks where Schur decomposition is applied to each non-overlapping blocks randomly. In this presented digital watermarking method, a novel LWT and DCT in combination with Schur decomposition for digital grayscale image watermarking is designed to get the benefits of hybridization of this combination to enhance imperceptibility and robustness against image processing and geometrical attacks, such as DCT provides energy compaction characteristics, LWT provides excellent frequency localization characteristic, and Schur decomposition as a matrix decomposition provides different elements which can be utilized for hiding the watermark into the highest energy element of the unitary matrix of Schur decomposition. To obtain the optimal trade-off between imperceptibility and robustness, the entropy analysis is utilized for getting the best scaling factor value by the average entropy of all non-overlapping blocks. For enhancement of the security of the presented method, a pseudo-random sequence generated by a random permutation function is utilized for selecting the blocks randomly to hide the watermark bits.

The paper is categorized as follows. In Section 2, the various terminologies such as DCT, LWT, Schur decomposition, random permutation function, and entropy analysis are explained. The discussions of the proposed digital watermarking scheme are given in Section 3. In Section 4, the experimental results and compared schemes are discussed whereas the conclusion is given in Section 5.

## Preliminaries

### Discrete cosine transform

Discrete cosine transform transforms multimedia information into a sum of cosine waves at several frequencies and implements quantization to compress multimedia information. DCT transforms multimedia information into a hierarchy of subbands in the transform domain. For hiding the watermark, the subband is selected based on the basis of its information content to achieve robustness against various image processing and geometrical attacks. DCT is utilized in several applications such as cryptography, compression, image processing, and watermarking [20].

**Lifting wavelet transform**

The lifting wavelet transform [21] is proposed to reduce computation time and memory requirement as the lifting scheme allows a fully in-place implementation of wavelet transform. LWT has been used in various applications like image compression, watermarking, pattern recognition, and image processing as a powerful scheme. LWT requires half the number of calculations as compared to traditional discrete wavelet transform and LWT permits us to perform reversible integer wavelet transform. The three steps of lifting wavelet transform are as follows:

1) Splitting: The primary signal s(k) is divided into even and odd non-overlapping samples: So(k) and Se(k).

$$So(k) = s(2k+1), Se(k) = s(2k) \quad (1)$$

2) Prediction: Utilizing even signal samples to predict odd signal samples and the two divided signals should be closely correlated. This step can be viewed as a high-pass filter processing. The difference can be given as follows:

$$D(k) = So(k) - P(Se(k)) \quad (2)$$

where P[.] denotes as the predict operator, while D(k) denotes as detail signal. The D(k) indicates the high-frequency component of the input signal s(k).

3) Updating: Introducing the operator of updating U[.], and using the detail signal D(k) for updating even samples Se(k). This step can be viewed as a low-pass filter processing. The approximate signal C(k) can be given as follows:

$$C(k) = Se(k) + U(D(k)) \quad (3)$$

where the C(k) indicates the low-frequency component of the input signal s(k).

**Schur decomposition**

One of the important tools in numerical linear algebra is Schur decomposition [22, 23] which is a major intermediate step in singular value decomposition. A real matrix A is decomposed by the Schur decomposition into $U$ and $D$ matrices, and its form can be presented as follows:

$$A = UDU^T \quad (4)$$

where U is a unitary matrix, whilst D is an upper triangular matrix that has the real eigenvalues on the Schur (·) and the diagonal represents the Schur decomposition function.

**Entropy analysis**

The entropy can be utilized for enumeration of how much data content is presented in the image. It depicts the number of randomness and uncertainty contained in the image. The more contained image information indicates to better its quality. Also, entropy can be named as Shannos's entropy, and the entropy of data calculates the uncertainty of a data source. Entropy of Shannon can be defined as follows [24, 25]:

$$E(x) = -\sum p(x) \log p(x) \quad (5)$$

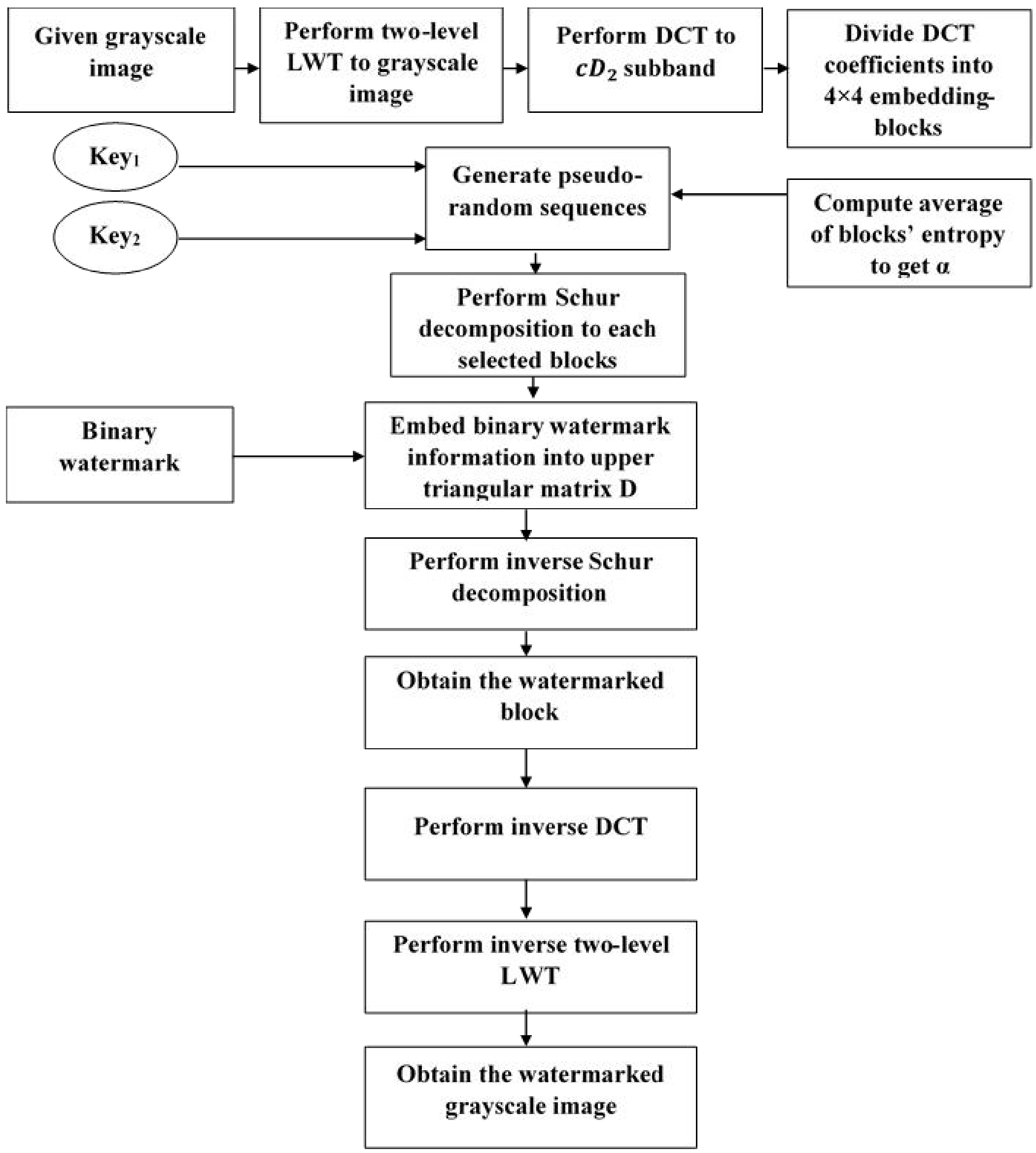


**Figure 1:** The proposed grayscale watermarking embedding process

## Proposed Scheme

A blind digital grayscale image watermarking method is proposed for hiding the watermark data into a grayscale image based on LWT and DCT in combination with Schur decomposition, and entropy analysis. In the proposed scheme, there are two processes: the watermark embedding process and the watermark extraction process as shown in figures 1 and 2 which are the block diagrams of these processes.

**Watermark embedding process**

The watermark embedding process can be given as below:

Step 1: A 2-level lifting wavelet transform is applied to the original image of size $N \times N$ for decomposition. For obtaining $cA_1$, $cH_1$, $cV_1$, and $cD_1$ of the first level then obtaining $cA_2$, $cH_2$, $cV_2$, and $cD_2$ of the $cD_1$ of the second level.

Step 2: The DCT is applied to the $cD_2$ high-frequency sub-band to obtain DCT coefficients.

Step 3: Splitting DCT coefficients for having non-overlapping blocks of size 4 × 4. In which the blocks' number is $64^2$.

Step 4: The entropy of each block is computed for having the average of blocks' entropy to use as quantization stage α.

Step 5: the pseudo-random sequences are generated by randperm-function based on the $key_1$ and $key_2$ to obtain two random numbers that indicate the row and column.

Step 6: Appling each selected 4 × 4 blocks based on the pseudo-random sequences by the Schur decomposition as mentioned in Eq. (4) to obtain the matrices $U$ and $D$.

Step 7: The magnitudes $M_1$ and $M_2$ are modified depending on the binary watermark data $w$, as below:

$$if\ w(i,j) = 1, \begin{cases} M_1 = 0.25\alpha \\ M_2 = -0.50\alpha \end{cases} \quad (6)$$

$$if\ w(i,j) = 0, \begin{cases} M_1 = -0.25\alpha \\ M_2 = 0.50\alpha \end{cases} \quad (7)$$

here ($1 \leq i, j \leq N$), and α indicates the quantization stage.

Step 8: Computing the potential quantization outcomes $T_1$ and $T_2$ based on the modified magnitudes $M_1$ and $M_2$.

$$T_1 = 2k\alpha + M_1 \quad (8)$$

$$T_2 = 2k\alpha + M_2 \quad (9)$$

here $k=floor(ceil(h_{1,1}/\alpha)/2)$, $ceil(.)$ and $floor(.)$ indicate as the largest nearest integer and the least nearest integer, respectively.

Step 9: The first-column, first-row component $d_{1,1}$ of upper triangular matrix $D$ is selected to hide the binary watermark as shown:

$$d'_{1,1} = \begin{cases} T_2\ if\ abs(d_{1,1} - T_2) < abs\ (d_{1,1} - T_1) \\ T_1\ else \end{cases} \quad (10)$$

here $abs(.)$ indicates the absolute value.

Step 10: The $d_{1,1}$ with $d'_{1,1}$ is replaced and then perform the inverse Schur decomposition to have the $A'$ watermarked block.

$$A' = UD'U^T \quad (11)$$

Step 11: Repeating steps 6–10 to hide all watermark data.

Step 12: Combination all watermarked blocks together.

Step 13: Applying the inverse of discrete cosine transform to the combined watermarked blocks to have the modified high-frequency sub-band $cD^w$, in which $cD^w$ indicates the watermarked estimated high-frequency sub-band.

Step 14: Applying the inverse of the 2-level lifting wavelet transform based on the changed high-frequency sub-band $cD^w$ instead of $cD_2$ for obtaining the watermarked grayscale image.

**Watermark extraction process**

The watermark extraction process can be given as below:

Step 1: A 2-level lifting wavelet transform is applied to the original image of size $N \times N$ for decomposition. For obtaining $cA_1$, $cH_1$, $cV_1$, and $cD_1$ of the first level then obtaining $cA_2$, $cH_2$, $cV_2$, and $cD_2$ of the $cD_1$ of the second level.

Step 2: The DCT is applied to the $cD_2$ high-frequency sub-band to obtain DCT coefficients.

Step 3: Splitting DCT coefficients for having non-overlapping blocks of size $4 \times 4$. In which the blocks' number is $64^2$.

Step 4: The entropy of each block is computed for having the average of blocks' entropy to use as quantization stage α.

Step 5: the pseudo-random sequences are generated by randperm-function based on the $key_1$ and $key_2$ to obtain two random numbers that indicate the row and column.

Step 6: Appling each selected 4 × 4 blocks based on the pseudo-random sequences by the Schur decomposition as mentioned in Eq. (4) to obtain the matrices $U$ and $D$.

Step 7: The first-column, first-row component $d_{1,1}$ of upper triangular matrix $D$ is used to extract the binary watermark as shown:

$$w'_{i,j} = mod\ (\ ceil\left(\frac{d'_{1,1}}{\alpha}\right), 2) \quad (12)$$

here ($1 \leq i, j \leq$N) and $mod(.)$ indicates the modulo operation.

Step 8: Repeating steps 6–7 to detect all watermark information and after that have the extracted binary watermark.

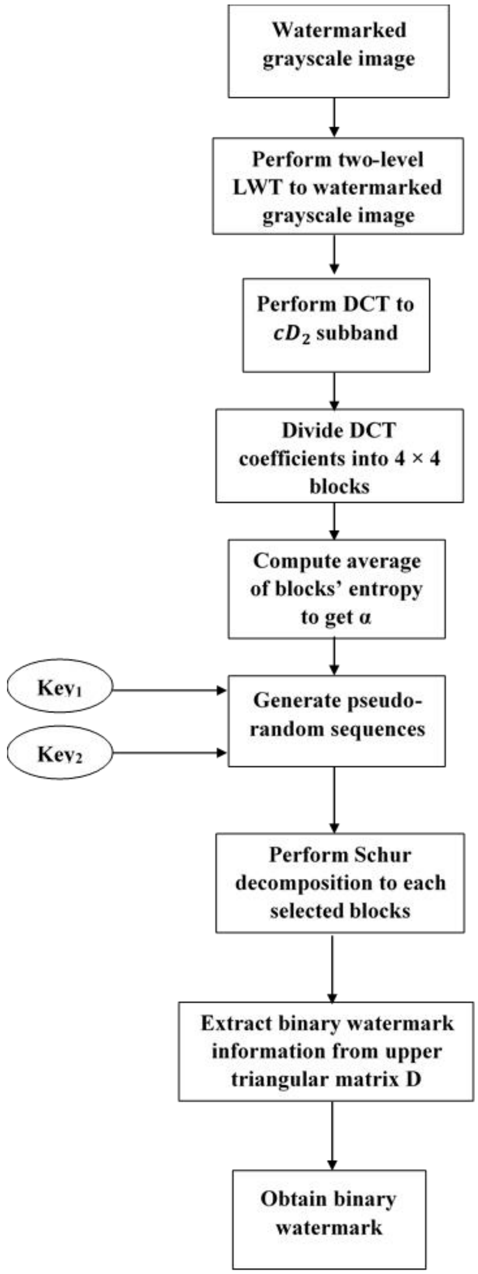


**Figure 2:** The proposed grayscale watermarking extraction process

## Experimental results and discussion

Performing several experimental results to present the effectiveness of the proposed grayscale image watermarking method in terms of invisibility and robustness to image processing and geometrical attacks. The experiment results were performed using MATLAB R2017b on a laptop whose classifications are Intel Core i3 2.00 GHz CPU, windows 10 environment, and 4.00 GB RAM. The proposed grayscale image watermarking method is applied to common pinchmarket grayscale images. Five pinch market grayscale images from the USC-SIPI database [26] are used in the proposed grayscale image watermarking method, the images are 1024 × 1024 as shown in figure 3 as referred to as “Lena”, “Sailboat”, “Peppers”, “Baboon”, and “Airplane, respectively. The watermarks used in this scheme of size 64×64 are shown in figure 4.

The attacks utilized in the proposed method are below:

- Flipping: Flipping the watermarked grayscale images up to down.
- Sharpening: Sharpening the watermarked grayscale images with 0.5 amount value.
- Gamma correction: Applying the Gamma correction to the watermarked grayscale images by 1.0 Gamma value.
- Blurring: Blurring the watermarked grayscale images with a 0.2 blurring value.
- Poisson noise: Adding the Poisson noise to the watermarked grayscale images.
- Scaling: Scaling the watermarked grayscale images to 8 times.
- Cropping: Cropping the watermarked grayscale images from the top 50%.

For evaluation of the invisibility and robustness of the proposed digital watermarking method, the peak signal-to-noise ratio (PSNR), and normalized cross-correlation (NC) are used to measure the ability of the proposed watermarking scheme to robust against attacks and the quality of the proposed watermarking method.

Peak signal-to-noise ratio is defined as an engineering phrase to calculate the ratio between the power of distorting noise and the highest potential power of a signal which impacts the fidelity of the signal's representation [27]. Its equation can be given as follows:

$$PSNR = 10 \log \frac{255^2}{MSE} dB \quad (13)$$

Here, MSE indicates the mean square error between the watermarked image and the cover image which is given as follows [28]:

$$MSE = \frac{1}{MN} \sum_{l=1}^{M} \sum_{k=1}^{N} (C(l,k) - C'(l,k))^2 \quad (14)$$

Here, *C(l,k)* and *C'(l,k)* indicate the *(l,k)*th pixel value in the cover image and watermarked Image, respectively.

Normalized cross-correlation is defined as a well-known metric used to measure the similarity degree between two compared images [29]. Its formula can be given as follows:

$$NC = \frac{\sum_{l=1}^{M} \sum_{k=1}^{N} W(l,k) \times W'(l,k)}{\sqrt{\sum_{l=1}^{M} \sum_{k=1}^{N} W^2(l,k)} \times \sqrt{\sum_{l=1}^{M} \sum_{k=1}^{N} W'^2(l,k)}} \quad (15)$$

Here, *W* and *W'* indicate the original watermark image and the extracted watermark image, respectively; after being attacked, *N* and *M* indicate the watermark image size.

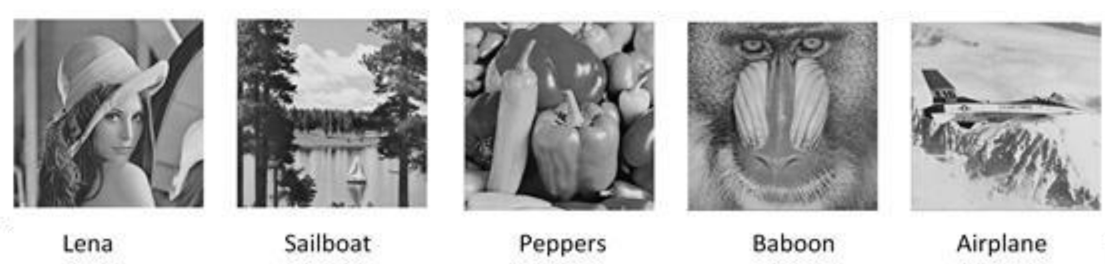


**Figure 3:** The grayscale original images


**Figure 4:** The watermark images

Tables 1 and 2 list the PSNR and NC values of the watermarked images with watermark images under no attacks. Generally, high PSNR values indicate that the proposed method offers superior performance in terms of invisibility, while high NC values indicate that the proposed method offers superior performance in terms of robustness. The PSNR values of the proposed image watermarking method are more than 45.570 dB while the NC values are more than 0.969.

**Table 1:** The PSNR values with no attacks

| Cover image | Watermark1 | Watermark2 | Watermark3 |
|---|---|---|---|
| **Lena** | 53.088 | 53.085 | 53.087 |
| **Sailboat** | 50.021 | 50.041 | 50.046 |
| **Peppers** | 52.101 | 52.117 | 52.107 |
| **Baboon** | 45.570 | 45.571 | 45.589 |
| **Airplane** | 54.703 | 54.699 | 54.696 |

**Table 2:** The NC values with no attacks

| Cover image | Watermark1 | Watermark2 | Watermark3 |
|---|---|---|---|
| **Lena** | 0.981 | 0.969 | 0.984 |
| **Sailboat** | 0.985 | 0.969 | 0.979 |
| **Peppers** | 0.985 | 0.975 | 0.981 |
| **Baboon** | 0.985 | 0.980 | 0.980 |
| **Airplane** | 0.984 | 0.970 | 0.987 |

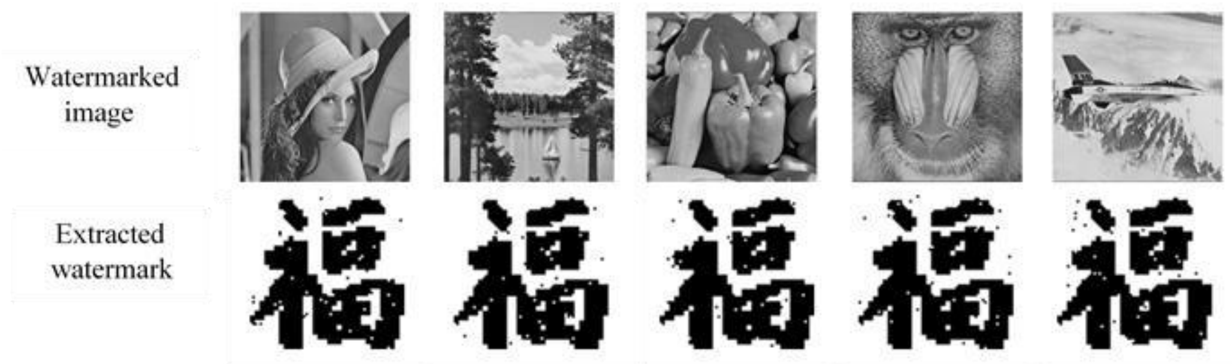


**Figure 5:** The watermarked images and extracted watermark 1 with no attack


**Figure 6:** The watermarked images and extracted watermark 2 with no attack


**Figure 7:** The watermarked images and extracted watermark 3 with no attack

In visual observations, figures 5 and 7 show the watermarked images with different watermark images under no attacks. As can be observed from table 2 and figures 5 to 7, the watermark images can be extracted fully and are more similar to the original watermark images which means the proposed method is more robust to attacks. Furthermore, as can be seen from tables 1 to 2 and figures 5 to 7, the watermarked images and the host images are more similar which means the proposed method is more invisible.

**Table 3:** The NC values of extracted watermark with watermark1 under different Attacks

| Attack | Lena | Sailboa | Pepper | Baboo | Airpla |
|---|---|---|---|---|---|
| **Flipping up to down** | 0.971 | 0.972 | 0.973 | 0.971 | 0.973 |
| **Sharpening (0.5)** | 0.903 | 0.907 | 0.908 | 0.895 | 0.904 |
| **Gamma correction (1.0)** | 0.981 | 0.985 | 0.985 | 0.985 | 0.984 |
| **Blurring (0.2)** | 0.962 | 0.964 | 0.970 | 0.967 | 0.967 |
| **Poisson noise** | 0.955 | 0.942 | 0.946 | 0.910 | 0.954 |
| **Scaling (8)** | 0.820 | 0.813 | 0.827 | 0.772 | 0.818 |
| **Cropping (50%)** | 0.884 | 0.890 | 0.888 | 0.883 | 0.888 |

Tables 3 to 5 list the NC values with watermark images from host images under various attacks whilst figures 8 to 10 observe the extracted watermark images from Lena image under different attacks in visual observations. As can be seen from these figures, the extracted watermark images are similar to the original watermark images and are extracted fully with a little distortion. It can be concluded from these tables and figures that the proposed image watermarking method is a superior performance in terms of both invisibility and robustness against attacks. For more investigation, the proposed image watermarking method is compared with the existing method that used the same original images and the same watermark image (watermark 1). Table 6 demonstrates the comparison of the proposed image watermarking method with this existing scheme [30]. As can be determined from this table, the proposed watermarking method has a superior performance in terms of invisibility than this existing scheme.

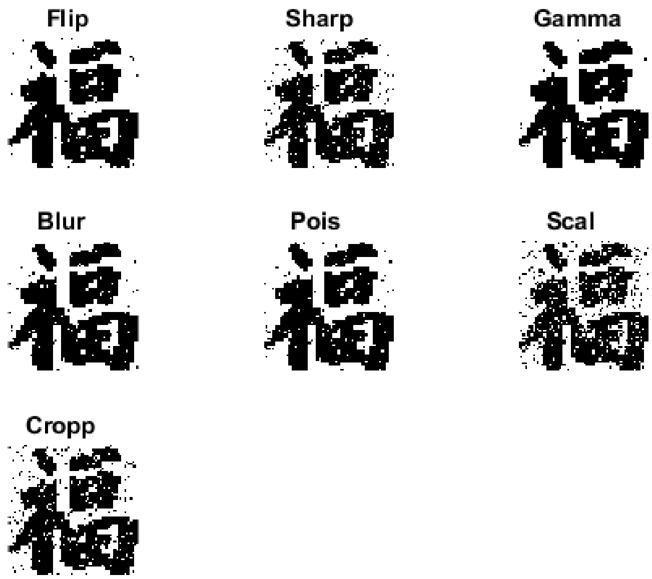


**Figure 8:** The extracted watermark 1 from Lena image under different attack

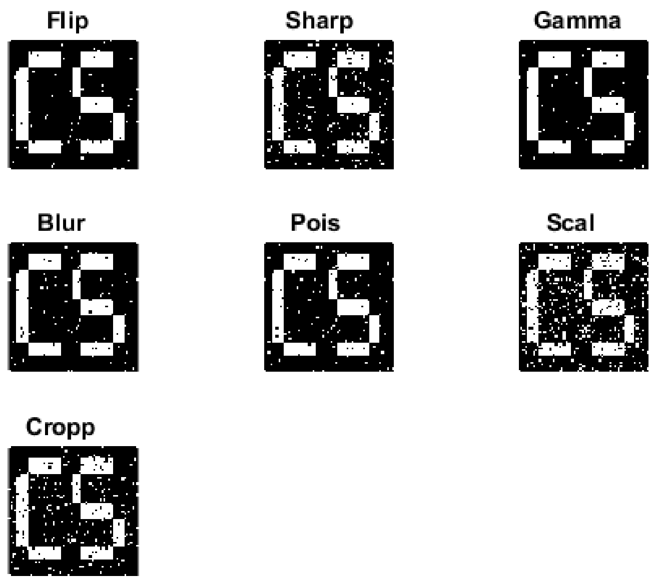


**Figure 9:** The extracted watermark 2 from Lena image under different attack

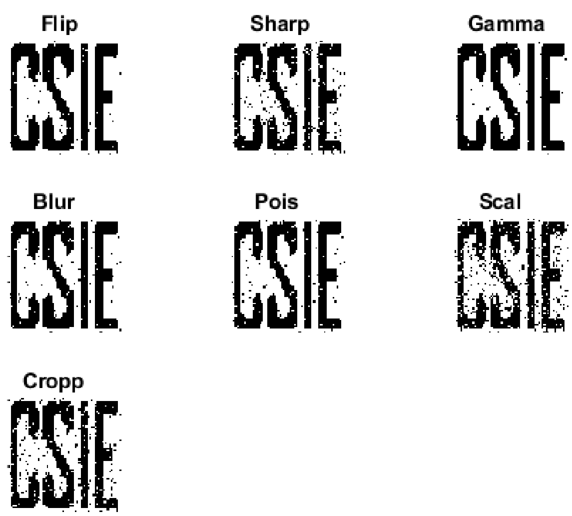


**Figure 10:** The extracted watermark 3 from Lena image under different attack

**Table 4:** The NC values of extracted watermark with watermark2 under different Attacks

| Attack | Lena | Sailboa | Pepper | Baboo | Airpla |
|---|---|---|---|---|---|
| **Flipping up to down** | 0.960 | 0.964 | 0.967 | 0.962 | 0.964 |
| **Sharpening (0.5)** | 0.874 | 0.876 | 0.871 | 0.853 | 0.864 |
| **Gamma correction (1.0)** | 0.969 | 0.969 | 0.975 | 0.980 | 0.970 |
| **Blurring (0.2)** | 0.946 | 0.942 | 0.953 | 0.951 | 0.943 |
| **Poisson noise** | 0.932 | 0.896 | 0.918 | 0.824 | 0.922 |
| **Scaling (8)** | 0.765 | 0.732 | 0.769 | 0.737 | 0.759 |
| **Cropping (50%)** | 0.858 | 0.850 | 0.866 | 0.845 | 0.854 |

**Table 5:** The NC values of extracted watermark with watermark3 under different Attacks

| Attack | Lena | Sailboa | Pepper | Baboo | Airpla |
|---|---|---|---|---|---|
| **Flipping up to down** | 0.973 | 0.970 | 0.980 | 0.976 | 0.966 |
| **Sharpening (0.5)** | 0.909 | 0.893 | 0.910 | 0.885 | 0.907 |
| **Gamma correction (1.0)** | 0.984 | 0.979 | 0.981 | 0.980 | 0.987 |
| **Blurring (0.2)** | 0.961 | 0.960 | 0.968 | 0.962 | 0.969 |
| **Poisson noise** | 0.956 | 0.943 | 0.952 | 0.881 | 0.950 |
| **Scaling (8)** | 0.822 | 0.808 | 0.813 | 0.784 | 0.812 |
| **Cropping (50%)** | 0.891 | 0.882 | 0.893 | 0.874 | 0.876 |

**Table 6:** The comparison of the proposed scheme of watermark 2 with the existing scheme

| Image | Scheme | PSNR |
|---|---|---|
| **Lena** | [30] | 45.731 |
| | Proposed | 53.088 |
| **Sailboat** | [30] | 43.796 |
| | Proposed | 50.021 |
| **Peppers** | [30] | 45.953 |
| | Proposed | 52.101 |
| **Baboon** | [30] | 45.682 |
| | Proposed | 45.570 |
| **Airplane** | [30] | 43.488 |
| | Proposed | 54.703 |

## Conclusion

A new robust blind image watermarking method is presented in this paper based on lifting wavelet transform, and Schur decomposition in combination with discrete cosine transform, and entropy analysis for copyright protection of multimedia information. In this method, various common pinch market images are used to hide various watermarks to analyze the effect of various watermarks on various host images. Various watermarking attacks are applied including flipping, Gamma correction, cropping, scaling, blurring, sharpening, and adding noise attacks. Two main properties imperceptibility and robustness in digital watermarking technology are obtained in this scheme with peak signal-to-noise ratio (PSNR) value with no attack > 45 dB, and normalize-cross-correlation (NC) value with no attack > 0.969. The proposed image watermarking method is invariant against common attacks with different images and different watermarks. The proposed image watermarking method has a superior performance in terms of imperceptibility and robustness.

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